

Fig. 4 Diagonal mean velocity profiles.

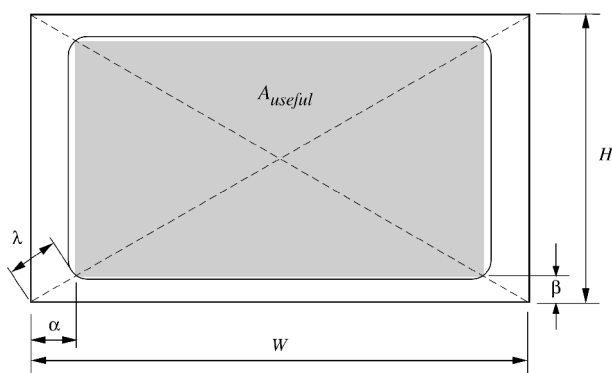


Fig. 5 Useful area of exit plane.

where  $\lambda$ ,  $\alpha$ , and  $\beta$  are shown in Fig. 5. The total cross-sectional area is  $A = WH$ , whereas the useful area is defined as  $A_{\text{useful}} = (W - 2\alpha)(H - 2\beta)$ . When Eq. (4) is used, the useful area becomes

$$A_{\text{useful}} = A(1 - 2C)^2 \approx 0.69A \quad (5)$$

Therefore, the estimated useful cross-sectional working area is a fixed proportion of the total cross-sectional area and has no dependence on AR. Likewise, conservative estimates for useful width and height are also independent of AR:  $W_{\text{useful}}/W = H_{\text{useful}}/H \approx 0.83$ .

#### IV. Conclusions

An experimental and computational analysis of the effect of changing the aspect ratio through a three-dimensional wind-tunnel contraction has been performed. Exit flow boundary layers were measured for four contractions ranging from AR 1 to 4, all with equivalent lengths and contraction ratios. Their development was related to the pressure history on the walls, as well as to the related effect of streamline convergence. Comparisons between contractions with different AR show that streamline convergence effects as well as pressure histories need to be considered when performing comparisons of boundary-layer thicknesses on the same wall of the contraction. For a given AR, the side wall boundary layer was found to be thicker than the floor boundary layer for  $AR > 1$ . The 45-deg corner boundary-layer thicknesses were found to decrease with increasing AR, consistent with the corner pressure distribution induced by the streamline convergence effects.

An examination of the mean velocity profile along the diagonal of the exit plane reveals a collapse of data. From this, an approximate design rule for the estimation of useful area in the working section has been proposed. For the contraction parameters considered in this study, the useful area is found to be  $0.69A$ , and this result is independent of AR.

#### Acknowledgments

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## New $\nu_t$ - $k$ Model for Calculation of Wall-Bounded Turbulent Flows

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#### Introduction

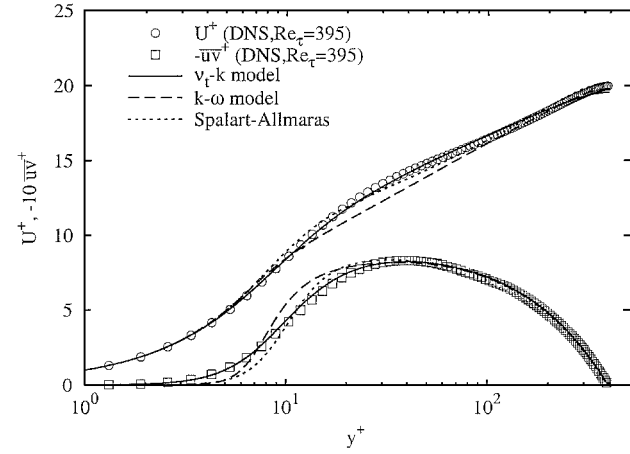
HISTORICALLY, the Boussinesq eddy viscosity that is assumed for Reynolds shear stress has been indirectly assessed by estimating the velocity scale and length scale of turbulent energetic eddies. Physical models to find these scales were the mixing length model,<sup>1</sup> the one-equation model employing the  $k$  equation of Bradshaw et al.,<sup>2</sup> and the two-equation model using the  $k$ - $\epsilon$  equations of Jones and Launder<sup>3</sup> or the  $k$ - $\omega$  equations of Wilcox.<sup>4</sup> More recently, the one-equation model has reemerged by Baldwin and Barth<sup>5</sup> and Spalart and Allmaras.<sup>6</sup> Instead of the  $k$  equation, however, they proposed to solve the  $\nu_t$  equation to assess directly the eddy viscosity. Subsequently, Durbin et al.<sup>7</sup> and Menter<sup>8</sup> formulated other forms of  $\nu_t$  equations.

To avoid the problem in the one-equation model that the length scale needs to be provided externally, these one-equation models use the von Kármán length scale,  $L_{vK} \equiv |S/\nabla S|$ , where  $S$  is the mean rate of strain, or a composite length scale,<sup>7</sup>  $(|\nabla \nu_t/S|)$ . Because of the advantages of directness of eddy viscosity assessment, simplicity in numerical computation, and use of natural boundary condition for the  $\nu_t$  equation, the one-equation model has often been a preferred choice in aerodynamic calculations. However, such one-equation models have the following three serious problems. One is that the length scale defined in any of the two methods mentioned earlier varies physically unrealistically in both the near-wall layer and the core region. The second problem is that predicted  $\nu_t$  varies as  $y^4$  as  $y \rightarrow 0$  (Fig. 1). The third problem is that when the one-equation

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a) Mean velocity and Reynolds stress

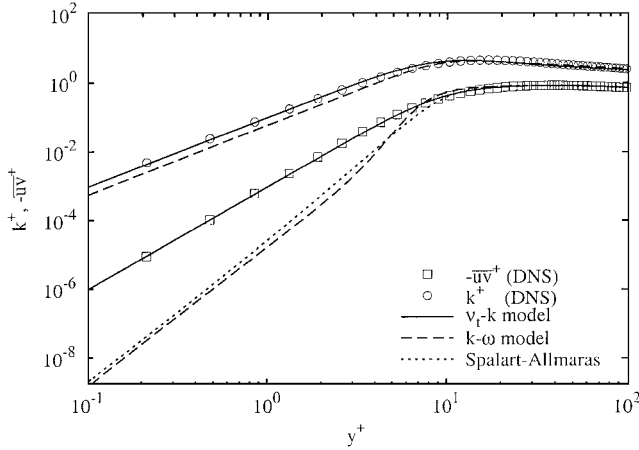
b) Log-log plot of  $k^+$  and  $-\overline{uv}^+$ 

Fig. 1 Fully developed channel flow.

model is applied to analyze the transitional boundary-layer problem, it is impossible to impose the freestream condition that affects the transition significantly. The aim of the present study is to formulate a new  $v_t$ - $k$  model that overcomes these problems of the one-equation model but keeps its advantages.

### Model Formulation

When a possibility that there were cross-diffusion terms in  $k$ - $\varepsilon$  and  $k$ - $\omega$  equations that are represented by products between gradients of variables  $k$ ,  $\varepsilon$ , or  $\omega$  (Ref. 9) is assumed, a method similar to that used by Menter<sup>8</sup> with an assumption  $\sigma_k = \sigma_\varepsilon$  reveals that the most general form of  $v_t$  equation can be written as

$$\begin{aligned} \frac{Dv_t}{Dt} = & C_{n1} \frac{v_t^2}{k} \left( \frac{\partial U}{\partial y} \right)^2 - C_{n2} k - C_{n3} \left( \frac{\partial v_t}{\partial y} \right)^2 + C_{n4} \frac{v_t}{k} \frac{\partial k}{\partial y} \frac{\partial v_t}{\partial y} \\ & + C_{n5} \frac{v_t^2}{k^2} \left( \frac{\partial k}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left[ \frac{v_t}{\sigma_n} \frac{\partial v_t}{\partial y} \right] \end{aligned} \quad (1)$$

In this equation, the fifth term with  $(\partial k / \partial y)^2$  on the right-hand side may be neglected due to its relative smallness in equilibrium and core layers in comparison with other terms. Now, there are five model constants,  $C_{n1}$ ,  $C_{n2}$ ,  $C_{n3}$ ,  $C_{n4}$ , and  $\sigma_n$ , to be determined. Because the low-Reynolds-number, turbulent kinetic energy equation in the following form<sup>3</sup> has been widely used and its validity has been well established by a direct numerical simulation (DNS) database, it is adopted in the present study without any modification:

$$\frac{Dk}{Dt} = v_t \left( \frac{\partial U}{\partial y} \right)^2 - C_\mu f_\mu \frac{k^2}{v_t} + \frac{\partial}{\partial y} \left[ \left( \nu + \frac{v_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] - 2\nu \left( \frac{\partial \sqrt{k}}{\partial y} \right)^2 \quad (2)$$

where  $\sigma_k = 1.0$  and  $C_\mu = 0.09$ . For a high Reynolds number, it is easy to show that the  $v_t$ - $k$  model leads to  $k \sim t^{-C_\mu/(C_\mu - C_{n2})}$  for shear-free homogeneous turbulence. Because  $k \sim t^{-1.2}$  for high-Reynolds-number turbulence, it is required that  $C_{n2} = C_\mu/6$ . For high-Reynolds-number homogeneous turbulence with uniform mean shear, the  $v_t$ - $k$  equations yields the following relation:

$$P_k/\varepsilon(1 - C_{n1}) - [1 - (C_{n2}/C_\mu)] = 0 \quad (3)$$

In a summary of 29 experiments on homogeneous uniform shear flow, Tavoularis and Karnik<sup>10</sup> showed that the ratio  $\varepsilon/P_k$  is nearly constant, and it is independent of the Reynolds number. Taking the mean value of 0.7 for  $\varepsilon/P_k$ ,  $C_{n1}$  is determined to be 0.42. Another relation between model constants is obtained from the log layer behavior,<sup>9</sup> which results in

$$C_{n1} - (C_{n2}/C_\mu) + (\kappa^2/\sqrt{C_\mu})[(1/\sigma_n) - C_{n3}] = 0 \quad (4)$$

where  $\kappa = 0.41$  is the von Kármán constant.

Now, we need two more relations to determine uniquely the model constants,  $C_{n3}$ ,  $C_{n4}$ , and  $\sigma_n$ . These can be found from two arguments: One is the study on the behavior of two-equation models at the edge of a turbulent region by Cazalbou et al.,<sup>11</sup> and the other is the asymptotic form of  $k$  in the free boundary edge of a shear-free turbulent mixing layer. With these guides, we obtained that  $C_{n3} = 0.95$ ,  $C_{n4} = 1.6$ , and  $\sigma_n = 2.0$ . These constants yield an asymptotic behavior  $k \sim y^{-2.78}$  at the edge of a shear-free mixing layer, which is a little steeper than experimental observation.<sup>12</sup> For the near-wall region, because  $k \sim y^2$  and  $v_t \sim y^3$  as  $y \rightarrow 0$ , the molecular diffusion term in Eq. (2) becomes the largest one as  $y \rightarrow 0$ , and it is necessary to make the budget in balance by damping the dissipation term with a damping function  $f_\mu$ . It is defined that  $f_\mu \equiv (\varepsilon - \varepsilon_w)/(C_\mu k^2/\nu_t)$ , and  $\varepsilon_w = 2\nu(\partial\sqrt{k}/\partial y)^2$ . Using DNS data of channel flow at low Reynolds number,<sup>13</sup>  $f_\mu$  is obtained as

$$f_\mu = 1 - \exp[-(R_y/A_1)^3] \quad (5)$$

where  $R_y = \sqrt{(ky)}/\nu$  and  $A_1 = 55$ . Next, consider Eq. (1) with a molecular diffusion term for the low-Reynolds-number region. Expand  $k$  and  $v_t$  near the wall as

$$k = b_k y^2 + c_k y^3 + \dots, \quad v_t = c_n y^3 + d_n y^4 + \dots \quad (6)$$

We find that the molecular diffusion term behaves near the wall as

$$\frac{\partial}{\partial y} \left[ \nu \frac{\partial v_t}{\partial y} \right] = \nu(6c_n y + 12d_n y^2) + \dots \quad (7)$$

which becomes the largest term for  $y \rightarrow 0$ . To counterbalance the budget near the wall, it is necessary to subtract a new term,

$$\frac{8}{3} \nu \left( \frac{\partial \sqrt{v_t}}{\partial y} \right)^2 = \nu(6c_n y + 10d_n y^2) + \dots \quad (8)$$

This term seems to contribute to the destruction of  $v_t$  by molecular viscosity. When this term is subtracted, the model equations yield too high a level of turbulent kinetic energy in the buffer layer, which is attributed to the next largest two terms that behave as  $y^2$  for  $y \rightarrow 0$ . Damping functions  $f_{n2}$  and  $f_{n4}$  are, therefore, introduced to these terms to reduce their magnitude near the wall. Numerical optimization is used to find them in the following forms:

$$f_{n2} = 1 - \exp[-(R_y/200)^4], \quad f_{n4} = 1 - \exp[-(R_y/100)^4] \quad (9)$$

Final equations of the proposed  $v_t$  equation is

$$\begin{aligned} \frac{Dv_t}{Dt} = & C_{n1} \frac{v_t^2}{k} \left( \frac{\partial U}{\partial y} \right)^2 - C_{n2} f_{n2} k - C_{n3} \left( \frac{\partial v_t}{\partial y} \right)^2 \\ & + C_{n4} f_{n4} \frac{v_t}{k} \frac{\partial v_t}{\partial y} \frac{\partial k}{\partial y} + \frac{\partial}{\partial y} \left[ \left( \nu + \frac{v_t}{\sigma_n} \right) \frac{\partial v_t}{\partial y} \right] - \frac{8}{3} \nu \left( \frac{\partial \sqrt{v_t}}{\partial y} \right)^2 \end{aligned} \quad (10)$$

Equations (2) and (10) constitute the new  $v_t$ - $k$  model. These equations with the selected model constants have been used to calculate

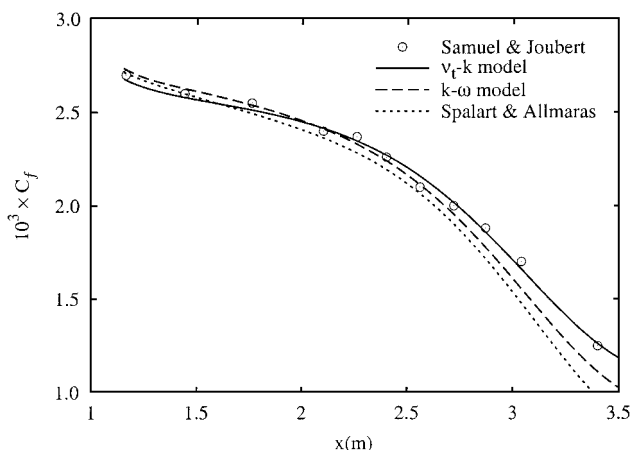


Fig. 2 Friction coefficient in Samuel-Joubert<sup>15</sup> flows.

a fully developed two-dimensional channel flow at  $Re_\tau = 395$  (Ref. 13), a flat plate boundary-layer flow,<sup>14</sup> and the boundary-layer flow under adverse pressure gradient.<sup>15</sup>

### Applications

Figure 1 shows the comparisons of the predicted  $U^+$ ,  $-\overline{uv}^+$ , and  $k^+$  using various models with DNS data. The one-equation model of Spalart and Allmaras<sup>6</sup> and the two-equation model of Wilcox<sup>9</sup> produce slightly larger Reynolds shear stress in the buffer layer, which results in lower velocity profiles in that region. It can be seen that the proposed  $v_t-k$  model yields correct near-wall behavior of the Reynolds shear stress and, thus, the eddy viscosity. Note that other models predict  $-\overline{uv}^+ \propto y^4$  as  $y \rightarrow 0$ . The present model is applied to a flat plate boundary layer, which results in good prediction of friction coefficients. A relation,  $l = \nu_t / \sqrt{k}$ , can infer the turbulence length scale from the  $v_t-k$  model, and it can be seen that the length scale  $l$  varies smoothly from zero at the wall to a value in the core region that is similar to that of the conventional algebraic model. For all models considered in this study, the computed skin friction was affected by the location of the first grid point,  $y_1^+$ . A comparison of such an influence between  $k-\varepsilon$  (Ref. 16) and  $v_t-k$  models revealed that a twice larger value of  $y_1^+$  can be used in the  $v_t-k$  model to produce the skin friction within  $\pm 1\%$  variation.

Finally, the adverse pressure gradient flow of Samuel and Joubert<sup>15</sup> has been computed. Figure 2 shows that the  $v_t-k$  model best predicts skin-friction coefficients in adverse pressure gradients flows. The predicted velocity and the Reynolds stress profiles, which are not shown here, were also comparable with the experiment.

### Conclusions

A new eddy viscosity transport equation is developed to constitute a new  $v_t-k$  model. Natural wall boundary conditions can be used for both  $v_t$  and  $k$  equations. Although this model is not free from defects due to the Boussinesq approximation, it yields favorable predictions of a number of standard attached turbulent boundary-layer flows. Further studies are in process to verify the present model for separated flows and compressible flows.

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## Structural Damage Identification Using Pole/Zero Dynamics in Neural Networks

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### I. Introduction

EFFECTIVE damage detection and/or identification of structure systems has been the subject of recent studies. The concept is based on that when a structure system undergoes different kinds or degrees of damage, its characteristics will change. A set of structural signatures can then be applied to characterize the damage condition(s). Structural damage identification by artificial neural networks has proven preferable over traditional model-based schemes because of the neural networks, advantages in self-learning, noise insensitivity, fault tolerance, and generalization ability. A feedforward neural network usually has three layers: the input layer, hidden layer, and output layer. From the viewpoint of pattern recognition, a diagnosis network that avoids the complicated algorithms in traditional model-based schemes can construct the mapping of the structural signatures and damage condition. In addition, once an artificial neural network is constructed, it can provide damage identification fast enough for online monitoring.

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